Contents

1 Rate and Ratio

Speed

Speed is the distance travelled by an object per unit time. i.e.,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}.$$

For example:

(i) A bus travels 120 km in 3 hours. The speed of the bus is

$$\left(\frac{120}{3}\right)\text{km/h} = 40 \text{ km/h}.$$

(ii) Kelvin runs 90 minutes at a speed of 8 km/h. The distance that he runs is

$$\left(8 \times \frac{90}{60}\right)\text{km} = 12 \text{ km}.$$

(iii) A runner runs 100 m at a speed of 10 m/s. The time that he runs is

$$\left(\frac{100}{10}\right)\text{s} = 10 \text{ s}.$$

> **Let's Bridge**
>
> 1. The distance between city A and city B is 350 km. If the time that a bus travels from one city to another city is 5 hours. Find the speed of the bus in km/h.
>
> 2. Sam walks 2.5 minutes at a speed of 1.5 m/s. Find the distance that he walks.
>
> 3. Simon rides a bicycle at a speed of 16 km/h. How long does he take to finish 20 km?

1. Rate

Rate（率）is a comparison of two different kinds of quantities and it carries units.

For example:

Billy can type 60 words in 3 minutes. The rate of his typing is

$$\left(\frac{60}{3}\right)\text{words/min} = 20 \text{ words / min}.$$

2. Ratio

Ratio（比）is a comparison of two or more quantities of the same kind and it carries no units.

For example:

Eddie finished a task in 4 weeks. Mary finished the same task in 42 days.

The ratio of time they needed $= 4 \times 7 : 42$

$$= 28 : 42$$

$$= 2 : 3 \text{ (or } \frac{2}{3} \text{)}$$

> **Attention !**
>
> The ratio can be expressed in the form:
>
> **(a)** $\dfrac{a}{b}$
>
> **(b)** $a : b$

3. Simplification of Ratios

Suppose $k \neq 0$，then

(a) $a : b = a \times k : b \times k$；

(b) $a : b = \dfrac{a}{k} : \dfrac{b}{k}$.

4. Ratio of Three Quantities

For example: If $a : b = 4 : 3$ and $b : c = 2 : 5$, find the ratio of $a : b : c$.

$$
\begin{array}{ccccccc}
a & : & b & & = & 4 & : & 3 & & \\
& & b & : & c & = & & 2 & : & 5 \\
\hline
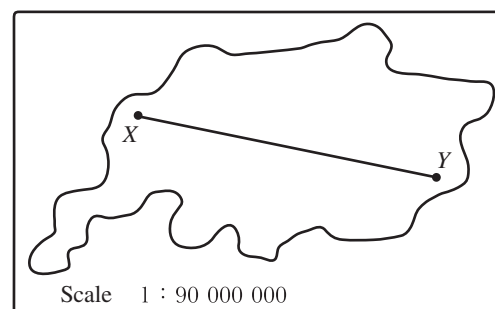a & : & b & : & c & = & 8 & : & 6 & : & 15
\end{array}
$$

5. Scale Drawings

(a) A **scale**（比例尺）in a map refers to the ratio of the length in the map to the actual length.

(b) It is usually expressed in the form $1 : n$, i.e., 1 cm in the map represents actual length of n cm.

(c) For example:

In the figure, $XY = 4$ cm. Find its actual length.

$$\frac{1}{90\ 000\ 000} = \frac{4 \text{ cm}}{\text{Actual length}}$$

$$= (4 \times 90\ 000\ 000)\text{cm}$$

Actual length $= 360\ 000\ 000$ cm

$$= 3\ 600\ 000 \text{ m}$$

$$= 3600 \text{ km}$$

Scale $1 : 90\ 000\ 000$

Conventional Questions

Level 1

1. A machine can produces 1200 toy cars in 8 hours.

(a) Find the production rate of the machine in toy cars / hour.

(b) Find the production rate of the machine in toy cars / second.

(c) Suppose the production rate of the machine remains unchanged. How many toy cars can the machine produce in 15 hours?

2. Determine whether a rate or a ratio should be used to represent the relation between the quantities.

(a) The melting points of aluminum and zinc are 660.25 °C and 419.73 °C respectively.

(b) The weight of 1000 paper clips is 1.2 kg.

3. Simplify the following ratios.

(a) $40 : 100$ **(b)** $3.6 : 5.4$ **(c)** $60 \text{ minutes} : 3\dfrac{3}{4} \text{ hours}$ **(d)** $\dfrac{1}{2} \text{ kg} : 1\dfrac{1}{4} \text{ g}$

4. Find $a : b : c$ in each of the following.

    (a) $a : b = 3 : 2$ and $b : c = 2 : 3$

    (b) $a : b = 4 : 3$ and $b : c = 1 : 5$

    (c) $a : b = 4 : 3$ and $b : c = 4 : 5$

5. The ratio of the heights of Anson to that of his father is $2 : 5$. The sum of their heights is 280 cm. Find the height of Anson.

6. The typing rate of Peter is 64 words / min. How many words can he type in 2.5 minutes?

7. The weight of Terry is heavier than Amy by 30 kg, and the weight of Jenny is heavier than Terry by 12 kg. If the weight of Terry is 63 kg, find the ratio of the weight of Amy to that of Jenny.

Level 2

8. Find $a : b : c$ in each of the following.

    (a) $a : b = 3 : 4$ and $a : c = 5 : 3$

    (b) $a = 3b$ and $b = 5c$

9. If $x : y : z = 2 : 3 : 4$ and $x + y + z = 27$, find the values of x, y and z.

10. The distance between locations A and B is 30 km. Katie walks half of the journey in 3 hours, and rides a bicycle at a speed of 20 km/h to finish another half of the journey. Find her average speed for the whole journey in km/h.

11. The figure shows the scale drawing of a plane. The length of the plane in the drawing is 9 cm. It is given that the actual length of the plane is 36 m.

    (a) Find the scale of the drawing. (Express the answer in the form $1 : n$.)

    (b) If the actual length of a window is 0.18 m, find its length in the drawing. (Express the answer in mm.)

9 cm
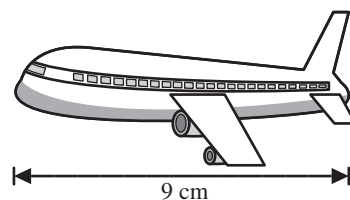
12. It is given that the ratio of the numbers of red marbles, blue marbles and green marbles is $2 : 3 : 5$. If there are 20 green marbles in the bag,

    (a) find the total number of marbles;

    (b) find the number of red marbles.

13. There is 3000 mL mixed liquid of alcohol and water. It is known that the volume of alcohol is 60 mL. If another 600 mL of alcohol is added in the mixed liquid, find the ratio of the volume of alcohol to the total volume of the mixed liquid.

14. The ratio of the length to the width of a badminton court is 11 : 5. If the length is 13.2 m, find the area of the badminton court.

13.2 m
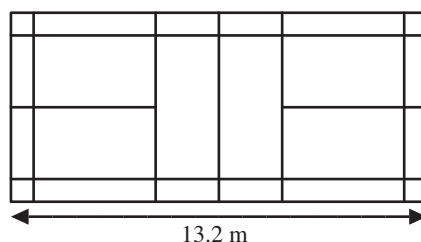
Level 3

15. Find $a : b : c$ in each of the following.

(a) $a : b = \dfrac{2}{3} : \dfrac{3}{5}$ and $b : c = \dfrac{1}{2} : \dfrac{3}{4}$

(b) $2a = 4b = 3c$

16. If x, y and z are non-zero numbers such that $2x = 3y$ and $x = 2z$, find $(x + y) : (x + z)$.

17. A total of \$144 is distributed to Sam, Terrence and Bobby. The money that Terrence gets is $\dfrac{1}{3}$ of Sam, and it is also equal to $\dfrac{1}{2}$ of Bobby. Who gets the most? How much does he get?

18. In a map, the distance between island A and island B is 6.5 cm. A ship travels from island A to island B directly at a speed of 38 km/h. If the scale of the map is 1 : 2 000 000, how long does it take for the whole journey? (Give the answer correct to 2 decimal places.)

19. 1 L of grapefruit honey is made by grapefruit pulp and water. The original ratio of grapefruit pulp to water is 1 : 9. In order to increase the concentration, Raymond decides to decrease the volume of water and increase the volume of grapefruit pulp, but keeps the total volume unchanged. It is given that the cost of grapefruit pulp is \$0.015/mL, while the cost of water is \$0.003/mL. In the new recipe, the ratio of the cost of grapefruit pulp to that of water is 5 : 3. Find the increased volume of grapefruit pulp.

20. City X and city Y are 60 km apart. The figure shows the graphs for car A and car B travelling on the same straight road between city X and city Y during the period 7:30 a.m. to 9:30 a.m. Car A travels at a constant speed during the period. Car B stays rest from 7:30 a.m to 8:15 a.m., and then travels at a constant speed until it arrives city Y.
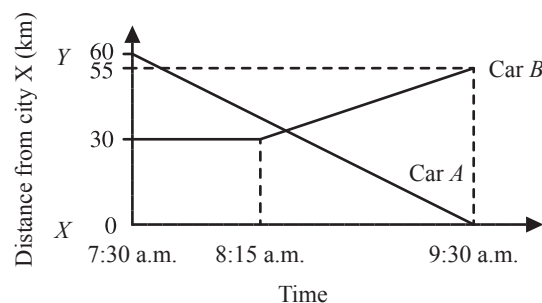
(a) Find the distance of car A from city X at 8:15 a.m.

(b) When does car B arrive city Y?

(c) At what time do car A and car B meet?

(d) The driver of car B claims that car B can travel longer distance than car A during the same travelling time. Do you agree? Explain your answer.

21. A telecommunication company provides a plan for long-distance call. In the plan, the fee of the call from Hong Kong to China are $0.5/min for the first 30 minutes, $0.4/min for the next 30 minutes, and $0.3/min for the rest of the time.

   (a) According to the following calling time, find the average fee charged by the company. (Express the answers in $/min.)

       (i) The calling time is 0.5 hour.

       (ii) The calling time is 1 hour.

       (iii) The calling time is 1.5 hours.

       (iv) The calling time is 2 hours.

   ♜ (b) Hence, what do you think about the charging policy? Explain your answer.

Multiple-choice Questions

Level 1

1. The volume of liquid A is 12L and it contains 20 g of mineral, how much mineral is contained in 30 L of liquid A?

   A. 25 g

   B. 40 g

   C. 50 g

   D. 56 g

2. Suppose a and b are non-zero numbers. If $4a + 3b = 2a + 7b$, then $a : b =$

   A. $2 : 1$.

   B. $1 : 2$.

   C. $5 : 3$.

   D. $3 : 5$.

3. If x, y and z are non-zero numbers such that $5x = 3y$ and $2x = 3z$, then $(x + y) : (x + z) =$

   A. $5 : 8$.

   B. $8 : 5$.

   C. $16 : 25$.

   D. $25 : 16$.

4. If 5 cm in a map represents actual length of 13 km, what is the scale of the map?

   A. $1 : 2600$

   B. $1 : 26\,000$

   C. $1 : 260\,000$

   D. $1 : 2\,600\,000$

5. The scale of floor plan of a flat is $1 : 150$. If the actual dimensions of the flat are 12 m × 8.1 m, find the area of the flat in the floor plan.

   A. $0.004\,32$ cm^2

   B. 0.432 cm^2

   C. 43.2 cm^2

   D. 4320 cm^2

6. Drink A and drink B are also mixed by lime juice and pineapple juice. The following table shows their ingredients.

	Ratio of lime juice to pineapple juice
Drink A	$3 : 2$
Drink B	$4 : 5$

   If 10 L of drink A is mixed with 18 L of drink B, find the ratio of the volume of lime juice to that of pineapple juice of the mixture.

   A. $1 : 1$

   B. $2 : 9$

   C. $3 : 2$

   D. $4 : 5$

For HKDSE
7. The cost of brand *A* rice is $56/kg, while the cost of brand *B* rice is $68/kg. 4 kg of brand *A* rice is mixed with 8 kg of brand *B* rice The cost of the mixture per kg is

A. $60.

B. $64.

C. $65.

D. $66.

Level 2

For HKDSE
8. Suppose 5 Great British Pounds (GBP) can be exchanged for 65 Hong Kong Dollars (HKD), and 2 Singapore Dollars (SGD) can be exchanged for 9 Hong Kong Dollars. How much GBP can be exchange for 100 SGD? Give the answer correct to the nearest GBP.

A. 585 GBP

B. 450 GBP

C. 52 GBP

D. 35 GBP

For HKDSE
9. If $a : b = 3 : 2$, $a : c = 4 : 3$ and $b : d = 5 : 4$, find $c : d$.

A. $9 : 7$

B. $5 : 8$

C. $9 : 8$

D. $45 : 32$

For HKDSE
10. If $\frac{1}{a} : \frac{1}{b} = 3 : 2$ and $a : c = 7 : 4$, then $a : b : c =$

A. $14 : 21 : 8$.

B. $21 : 14 : 8$.

C. $14 : 21 : 12$.

D. $21 : 14 : 12$.

11. If m and n are non-zero numbers such that $\frac{8m - 4n}{2m - n} = 3$, then $m : n =$

A. $2 : 1$.

B. $1 : 2$.

C. $5 : 3$.

D. $3 : 5$.

For HKDSE
12. The cost of tea *A* and tea *B* are $12/kg and $6/kg respectively. If x kg of tea *A* and y kg of tea *B* are mixed so that the cost of the mixture is $8/kg, find $x : y$.

A. $1 : 2$

B. $2 : 1$

C. $1 : 3$

D. $3 : 1$

13. In a test, the ratio of the marks of James to that of Dickson is 9 : 7. If the marks of James is 15 higher than Dickson, what is the marks of Dickson?

A. 22.5

B. 37.5

C. 52.5

D. 67.5

Cross-topics Challenge

1. The length, width and height of a rectangular container are 6 m, 4 m and 3 m respectively. If water is filled into the container at a rate of 1.5 m³/ min, how long should it take to make the container half full?

2. It is given that the ratio of three numbers is 5 : 7 : 9, and the average of the three numbers is 49. Find the smallest number.

3. The figure shows three squares *ABCD*, *AEFG* and *AHIJ*, where
$AB : BE : EH = 3 : 4 : 2$.

    (a) Find the ratio of the perimeters of the three squares.

    (b) **(i)** Find the ratio of the areas of the three squares.

         (ii) If the area of *AHIJ* is 252 cm^2, find the area of *ABCD*.
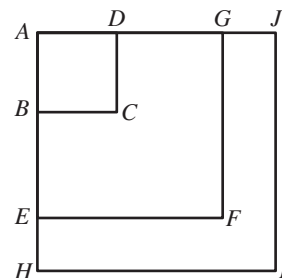
4. In the figure, it is given that $\triangle ABC \sim \triangle PQR$.
Find the values of *x* and *y*.
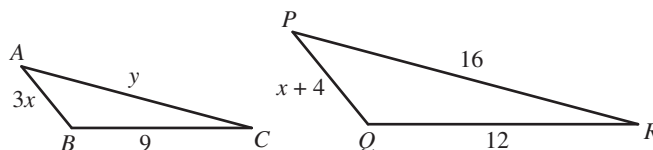
Common Tricky Questions

1. It is given that $\dfrac{5a}{3a + 2b} = \dfrac{1}{2}$, where *a* and *b* are non-zero numbers.
Find $(2a + b) : (b - a)$.   (4 marks)

$$\frac{5a}{3a + 2b} = \frac{1}{2}$$

$$5a \times 2 = 3a + 2b \qquad \text{[1M]}$$

$$10a - 3a = 2b$$

$$7a = 2b$$

$$\frac{a}{b} = \frac{2}{7} \qquad \text{[1A]}$$

$$\boxed{a = 2 \text{ and } b = 7}$$

$$\frac{2a + b}{b - a} = \frac{2 \times 2 + 7}{7 - 2}$$

$$= \frac{11}{5}$$

$$\therefore (2a + b) : (b - a) = \underline{11 : 5}$$

Examination Analysis

Beware that $\dfrac{a}{b} = \dfrac{2}{7}$ ($a : b =$
2 : 7) only shows their ratio
but not their actual values.
Therefore, we cannot directly
write that $a = 2$ and $b = 7$.

The correct way is:

Let $a = 2k$ and $b = 7k$, where *k*
is a non-zero number.

$$\therefore \frac{2a + b}{b - a} = \frac{2 \times 2k + 7k}{7k - 2k} \qquad \text{[1M]}$$

$$= \frac{11k}{5k} = \frac{11}{5}$$

$$\therefore (2a + b) : (b - a) = \underline{11 : 5} \quad \text{[1A]}$$

2. It is given that $3a = 5b$ and $2b = 5c$. Find $a : b : c$.   (3 marks)

$a : b = 3 : 5$ and $b : c = 2 : 5$

$$
\begin{array}{ccccccc}
a & : & b & & = & 3 & : & 5 & & \\
& & b & : & c & = & & 2 & : & 5 \\
\hline
a & : & b & : & c & = & 6 & : & 10 & : & 25
\end{array}
$$

Examination Analysis

Note that $3a = 5b$ does not imply $a : b$
$= 3 : 5$, it implies $\dfrac{a}{b} = \dfrac{5}{3}$, i.e., $a : b = 5 : 3$.

Similarly, we have $b : c = 5 : 2$.

The correct way is:

$a : b = 5 : 3$ and $b : c = 5 : 2$ [1M]

$$
\begin{array}{ccccccc}
a & : & b & & = & 5 & : & 3 & & \\
& & b & : & c & = & & 5 & : & 2 \\
\hline
a & : & b & : & c & = & 25 & : & 15 & : & 6
\end{array}
$$
    [1M + 1A]