Contents

10 Trigonometry

Smart Review

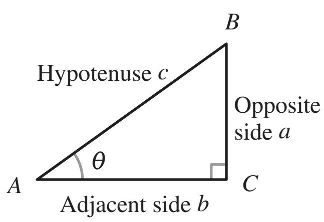
A. Trigonometric Ratios

1. In the figure, ABC is a right-angled triangle, where a is the **opposite side**（對邊）of θ, b is the **adjacent side**（鄰邊）of θ, c is the **hypotenuse**（斜邊）of θ. The trigonometric ratios of **sine**（正弦）, **cosine**（餘弦）and **tangent**（正切）are defined as follows.

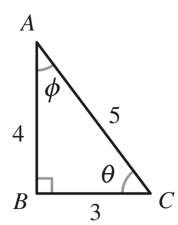
(a) $\sin\theta = \dfrac{\text{Opposite side of }\theta}{\text{Hypotenuse of }\theta} = \dfrac{a}{c}$

(b) $\cos\theta = \dfrac{\text{Adjacent side of }\theta}{\text{Hypotenuse of }\theta} = \dfrac{b}{c}$

(c) $\tan\theta = \dfrac{\text{Opposite side of }\theta}{\text{Adjacent side of }\theta} = \dfrac{a}{b}$



For example:



(i) In the figure, find the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

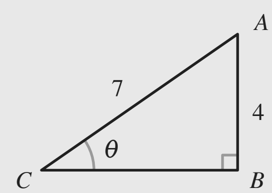
$\sin\theta = \dfrac{4}{5}$, $\cos\theta = \dfrac{3}{5}$, $\tan\theta = \dfrac{4}{3}$.

(ii) In the figure, find the values of $\sin\phi$, $\cos\phi$ and $\tan\phi$.

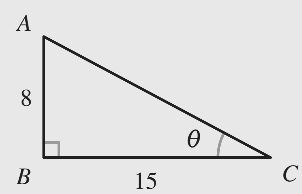
$\sin\phi = \dfrac{3}{5}$, $\cos\phi = \dfrac{4}{5}$, $\tan\phi = \dfrac{3}{4}$.

Let's Try

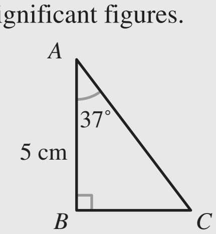
1. In the figure, find θ, correct to 1 decimal place.



2. In the figure, find the value of $\cos\theta$.



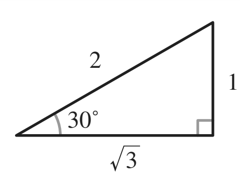
3. In the figure, find the length of BC, correct to 3 significant figures.

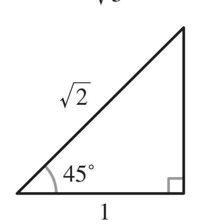


2. Trigonometric ratios of special angles

Referring to the two right-angled triangles on the right.

θ / Trigonometric ratio	30°	45°	60°
$\sin\theta$	$\dfrac{1}{2}$	$\dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{2}\right)$	$\dfrac{\sqrt{3}}{2}$
$\cos\theta$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{2}\right)$	$\dfrac{1}{2}$
$\tan\theta$	$\dfrac{1}{\sqrt{3}}\left(\dfrac{\sqrt{3}}{3}\right)$	1	$\sqrt{3}$





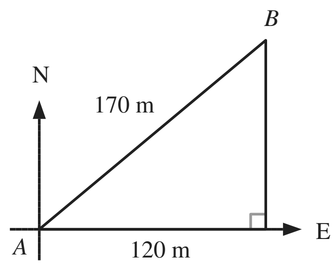
Worked Examples

Conventional Questions

Section A(1)

Reference: HKCEE 04 I Q5

1. In the figure, find the bearing of B from A. (Give the answer correct to 3 significant figures if necessary.) (3 marks)

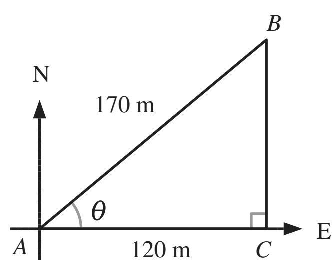


Mind the Trap

Candidates should let the required angle be θ before doing the calculation. Otherwise, 1 mark will be deducted for undefined symbol.

Solution

Let $\angle BAC = \theta$.



$\cos\theta = \dfrac{120}{170}$ [1M]

$\theta \approx 45.1^\circ$ (*cor. to 3 sig. fig.*) [1A]

$\because 90^\circ - 45.1^\circ = 44.9^\circ$

$\therefore$ The bearing of B from A is N44.9°E. [1A]

Solving Strategy

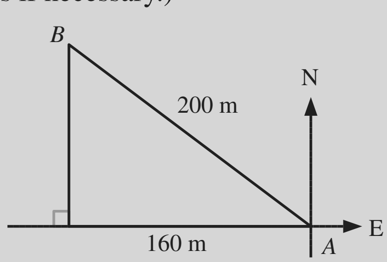
- Candidates should only round off the numerical answer to the required significant figures in the final step.
- The question requires to find the bearing of 'B from A', thus mark a cross at A.

Mind the Trap

Since no specific bearing is mentioned in the question, candidates can give 044.9° or N44.9°E as the answer.

Instant Drill

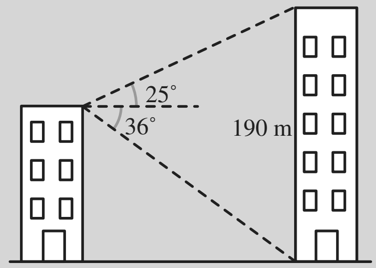
In the figure, find the bearing of A from B. (Give the answer correct to 3 significant figures if necessary.) (3 marks)



Instant Drill

In the figure, Katy measured the angle of elevation of the top of a building from the top of another building as 25°. She also measured the angle of depression of the bottom of the same building as 36°. The height of the measured building is 190 m. Find the distance between the two buildings. (Give the answer correct to 3 significant figures.)

(4 marks)



Multiple-choice Questions

Section A

Reference: HKCEE 08 II Q23

6. $\dfrac{\tan(90° - A)}{\cos A} =$

A. $\sin A$

B. $\cos A$

C. $\dfrac{1}{\sin A}$

D. $\dfrac{1}{\cos A}$

Solution

$$\frac{\tan(90° - A)}{\cos A}$$
$$= \frac{1}{\tan A \cos A}$$
$$= \frac{\cos A}{\sin A} \times \frac{1}{\cos A}$$
$$= \frac{1}{\sin A}$$

The answer is C.

MC Shortcut

Alternative Method (for multiple-choice questions only):
Set any value for A and put it into each expression and then compare the results. For example, let $A = 20°$, then the value of the given expression is 2.923….
Option A: 0.342…. ✘
Option B: 0.939…. ✘
Option C: 2.923…. ✔
Option D: 1.064…. ✘

Instant Drill

$\dfrac{\sin(90° - A)}{\tan(90° - A)} =$

A. $\sin A$

B. $\cos A$

C. $\dfrac{1}{\sin A}$

D. $\dfrac{1}{\cos A}$

Mock Questions

(In the following questions, unless otherwise specified, give the answer correct to 3 significant figures if necessary.)

Conventional Questions

Section A(1)

1. Simplify $\dfrac{\cos\theta}{\tan(90^\circ-\theta)}$. (2 marks)

2. Simplify $\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}$. (2 marks)

3. Simplify $2\sin(90^\circ-\theta)\cos 30^\circ-\cos\theta$. (3 marks)

4. Simplify $\dfrac{\cos(90^\circ-\theta)\tan(90^\circ-\theta)}{\cos\theta}$. (3 marks)

5. Simplify $\dfrac{\cos(90^\circ-\theta)}{\tan\theta}$. (3 marks)

6. Simplify $(1+\sin\theta)[1-\cos(90^\circ-\theta)]$. (3 marks)

7. Simplify $1-\dfrac{1}{\sin^2(90^\circ-\theta)}$. (3 marks)

8. Simplify $\dfrac{\sin 30^\circ}{1-\sin(90^\circ-\theta)}-\dfrac{\sin 30^\circ}{1+\sin(90^\circ-\theta)}$. (4 marks)

9. Prove that $\dfrac{\tan(90^\circ-\theta)}{\sin(90^\circ-\theta)}\equiv\dfrac{1}{\sin\theta}$. (3 marks)

10. Prove that $\sin^2\theta+\cos^2(90^\circ-\theta)\tan^2(90^\circ-\theta)\equiv 1$. (3 marks)

11. Prove that $\dfrac{1-2\sin^2\theta}{\cos\theta+\sin\theta}\equiv\cos\theta-\sin\theta$. (3 marks)

12. Without using a calculator, find the value of $\dfrac{\cos^2 45^\circ}{\sin^2 30^\circ-\tan^2 60^\circ}$. (4 marks)

13. Without using a calculator, find the value of $\dfrac{2\tan 45^\circ-\sin^2 45^\circ}{\tan^2 30^\circ}$. (4 marks)

Multiple-choice Questions
Section A

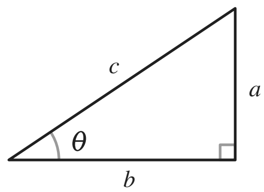
28. In the figure, $\cos\theta + \tan\theta =$

A. $\dfrac{a}{c} + \dfrac{a}{b}$.

B. $\dfrac{a}{c} + \dfrac{b}{a}$.

C. $\dfrac{b}{a} + \dfrac{b}{a}$.

D. $\dfrac{b}{c} + \dfrac{a}{b}$.



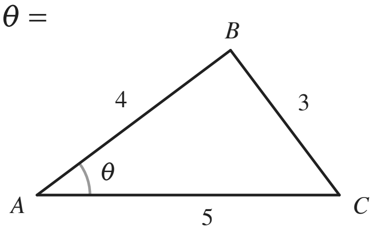
29. In the figure, $\tan\theta =$

A. $\dfrac{3}{4}$.

B. $\dfrac{3}{5}$.

C. $\dfrac{4}{5}$.

D. $\dfrac{5}{4}$.



Reference: HKCEE 08 II Q24

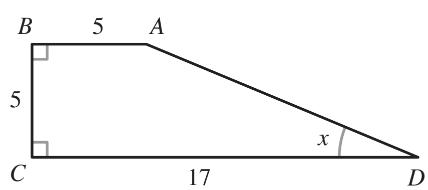
30. In the figure, $\sin x =$

A. $\dfrac{12}{17}$.

B. $\dfrac{12}{13}$.

C. $\dfrac{5}{13}$.

D. $\dfrac{5}{12}$.



Reference: HKCEE 06 II Q23

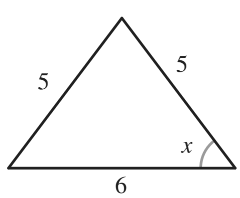
31. In the figure, $\sin x =$

A. $\dfrac{4}{3}$.

B. $\dfrac{3}{4}$.

C. $\dfrac{3}{5}$.

D. $\dfrac{4}{5}$.



Reference: HKCEE 04 II Q22

32. $\dfrac{\sin 30^\circ}{1 - \cos(90^\circ - \theta)} - \dfrac{\cos 60^\circ}{1 + \cos(90^\circ - \theta)} =$

A. $\dfrac{1}{\cos^2\theta}$

B. $\dfrac{\cos\theta}{\tan\theta}$

C. $\dfrac{\tan\theta}{\cos\theta}$

D. $\dfrac{1}{\cos\theta\tan\theta}$

Reference: HKDSE 12 II Q19

33. If A and B are both acute angles and $A + B = 90^\circ$, then $\dfrac{1}{\cos^2 A + \sin^2 B} =$

A. 1.

B. $\dfrac{1}{2\sin^2 A}$.

C. $\dfrac{1}{2\cos^2 A}$.

D. $\dfrac{1}{2\cos^2 B}$.

Reference: HKCEE 09 II Q24

34. $\dfrac{\tan(90^\circ - A)}{\cos A} =$

A. $\sin A$

B. $\cos A$

C. $\dfrac{1}{\sin A}$

D. $\dfrac{1}{\cos A}$

Reference: HKCEE 08 II Q23

35. If $0^\circ < x < 90^\circ$, which of the following must be correct?

I. $\sin x + \sin(90^\circ - x) > 0$

II. $\cos x \div \cos(90^\circ - x) > 1$

III. $\tan x \times \tan(90^\circ - x) = 1$

A. I only

B. II only

C. I and III only

D. II and III only

Reference: HKDSE 13 II Q23

Revision Test

Marks: ______ /100

Date: ______

Conventional Questions

1. Simplify $\dfrac{(x^2y^{-3})^5}{x^4y^{-2}}$ and express the answer with positive indices. (3 marks)

2. Make c as the subject of the formula $\dfrac{3+d}{1-2c}=5d$. (3 marks)

3. Factorize

(a) $4x^2-12xy+9y^2$,

(b) $4x^2-12xy+9y^2-2x+3y$.

(3 marks)

4. The cost of a watch is \$ 1200. If the watch is sold at a discount of 20% of its marked price, the profit percentage is 30%. Find the marked price. (4 marks)

5. The ratio of the costs of a bottle of orange juice to a bottle of milk is 5 : 3. If the total cost of 4 bottles of orange juice and 6 bottles of milk is \$76, find the cost of a bottle of milk. (4 marks)

6. In a polar coordinate system, the polar coordinates of points A, B and C are $(8, 123°)$, $(7, 213°)$ and $(6, 303°)$ respectively.

(a) Let O be the pole. Are A, O and C collinear? Explain your answer.

(b) Find the area of $\triangle ABC$.

(4 marks)

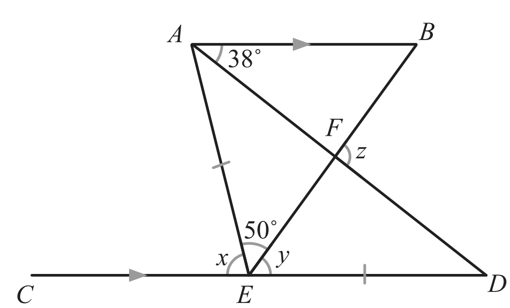
7. **(a)** Solve $\dfrac{2x-21}{5}\leq 4x+9$.

(b) Find the number of negative integers that satisfy the inequality in **(a)** and write down the smallest one.

(4 marks)

8. In the figure, E is a point on CD such that $AE = ED$. Given that AB // CD, $\angle BAD = 38°$ and $\angle AEB = 50°$. Find x, y and z.

(4 marks)



4 Identities and Factorization

Let's Try (p.52)

1. L.H.S. $= 3(5x - 2) - 4x$

$= 11x - 6$

R.H.S. $= 2(6x - 5)$

$= 12x - 10$

$\because$ L.H.S. $\neq$ R.H.S.

$\therefore 3(5x - 2) - 4x = 2(6x - 5)$ is not an identity.

2. L.H.S. $= (x + 1)(x + 3)$

$= x^2 + x + 3x + 3$

$= x^2 + 4x + 3$

$=$ R.H.S.

$\therefore (x + 1)(x + 3) = x^2 + 4x + 3$ is an identity.

Let's Try (p.52)

1. Comparing the like terms and the constant terms on the two sides, we have

$a = -4$ and $b = 7$.

2. Comparing the like terms and the constant terms on the two sides, we have

$3t = -9$ and $s + t = 6$,

i.e., $t = -3$ and $s = 9$.

> **Guidelines**
> As the calculation involved is easier, frst find the value of t. Then use the result to find s.

Let's Try (p.53)

1. $(3 + h)^2$

$= 3^2 + 2(3)h + h^2$

$= \underline{\underline{h^2 + 6h + 9}}$

> **Guidelines**
> Solutions are usually expressed in descending order of the variable.

2. $(16 - 7y)(16 + 7y)$

$= 16^2 - (7y)^2$

$= \underline{\underline{256 - 49y^2}}$

3. $(5m - 4)(25m^2 + 20m + 16)$

$= (5m - 4)[(5m)^2 + (5m)(4) + 4^2]$

$= (5m)^3 - 4^3$

$= \underline{\underline{125m^3 - 64}}$

Let's Try (p.53)

1. $2ab^2 - 4b = \underline{\underline{2b(ab - 2)}}$

2. $6x^2y^3z + 18xy^2z^2 - 9y^3z$

$= \underline{\underline{3y^2z(2x^2y + 6xz - 3y)}}$

> **Guidelines**
> The common factor of 6, 18 and 9 is 3, while the common factor of x^2y^3z、xy^2z^2 and y^3z is y^2z.

Let's Try (p.53)

1. $5p + 5q + 2mp + 2mq$

$= 5(p + q) + 2m(p + q)$

$= \underline{\underline{(p + q)(5 + 2m)}}$

2. $h^2 - 3jk - 3hj + hk$

$= h^2 + hk - 3jk - 3hj$

$= h(h + k) - 3j(k + h)$

$= \underline{\underline{(h + k)(h - 3j)}}$

Let's Try (p.54)

1.

$$\begin{array}{ccc} a & \times & -5 \\ a & & +1 \\ \hline -5a & & +a = (-5 + 1)a = -4a \end{array}$$

$\therefore a^2 - 4a - 5 = \underline{\underline{(a - 5)(a + 1)}}$

2.

$$\begin{array}{ll} 2x & -1 \\ 3x & +1 \\ \hline -3x & +2x = (-3+2)x = -x \end{array}$$

$\therefore 6x^2 - x - 1 = \underline{\underline{(2x-1)(3x+1)}}$

3.

$$\begin{array}{ll} 2m & +3n \\ 2m & +5n \\ \hline +6mn & +10mn = (6+10)mn = 16mn \end{array}$$

$\therefore 4m^2 + 16mn + 15n^2 = \underline{\underline{(2m+3n)(2m+5n)}}$

Let's Try (p.54)

1. $a^2 + 8a + 16$

$= a^2 + 2(4)a + 4^2$

$= \underline{\underline{(a+4)^2}}$

2. $u^2 - 10uv + 25v^2$

$= u^2 - 2(5v)u + (5v)^2$

$= \underline{\underline{(u-5v)^2}}$

3. $9m^2 - 49n^2$

$= (3m)^2 - (7n)^2$

$= \underline{\underline{(3m+7n)(3m-7n)}}$

4. $x^3 - 8$

$= x^3 - 2^3$

$= (x-2)[x^2 + x(2) + 2^2]$

$= \underline{\underline{(x-2)(x^2+2x+4)}}$

Common Mistakes

Some candidates may confuse the identity of sum of two cubes with that of difference of two cubes. In each of these two identities, there is only one negative sign. In the identity of difference of two cubes, the first +/– sign is negative; while in the identity of sum of two cubes, the second +/– sign is negative.

5. $27k^3 + 1$

$= (3k)^3 + 1^3$

$= (3k+1)[(3k)^2 - (3k)(1) + 1^2]$

$= \underline{\underline{(3k+1)(9k^2-3k+1)}}$

Concept Builder (p.55)

1. False

If an equation holds for 'any values' of the unknowns, then the equation is an identity.

2. False

L.H.S. $= 7(2x-1) + 8$

$= 14x + 1$

R.H.S. $= 8x + 6(x-2) + 5$

$= 14x - 7$

$\because$ L.H.S. $\neq$ R.H.S.

$\therefore 7(2x-1) + 8 = 8x + 6(x-2) + 5$ is not an identity.

3. False

In any identity, besides the constant terms, the like terms on the two sides are equal.

4. False

$(7x+10)^2 = (7x)^2 + 2(7x)(10) + 10^2$

$= 49x^2 + 140x + 100$

5. True

6. True

$4 - 4x = 4(1-x)$

$= -4(x-1)$

7. False

$$\begin{array}{ll} x & -4 \\ 3x & +5 \\ \hline -12x & +5x = (-12+5)x = -7x \end{array}$$

This method can be used to factorize $3x^2 - 7x - 20$ only.