Unit 1

Rate, Ratio and Proportion

Concept Review

1. Rate

Rate is a comparison of two different kinds of quantities and it carries units.

For example, a car travels 100 km in 2 hours, then its speed = $\frac{100}{2}$ km/h = 50 km/h.

2. Ratio

Ratio is a comparison of two or more quantities of the same kind and it carries no units.

For example, there are 8 red balls and 12 green balls in a box, then

the ratio of number of red balls to that of green balls = 8 : 12 = 2 : 3.

3. Continued Ratio

If $a : b = 2 : 3$ and $b : c = 6 : 5$, then

$$\begin{array}{l} a : b \quad = 2\cdot 2 : 3\cdot 2 \\ \quad\ \ b : c = \qquad\quad\ \ 6 : 5 \\ \hline a : b : c = 4 : 6 : 5 \end{array}$$

4. Scale drawing

For a map with a scale of 1 : 400 000, a length of 1 cm on the map represents an actual distance of 400 000 cm (i.e. 4 km).

5. Proportion

(a) Direct proportion

If A and B are in direct proportion, then

$$a_1 : a_2 = b_1 : b_2 \left(\text{or } \frac{a_1}{a_2} = \frac{b_1}{b_2} \right)$$

where a_1, a_2 are any two values of A and b_1, b_2 are the corresponding values of B.

(b) Inverse proportion

If A and B are in inverse proportion, then

$$a_1 : a_2 = b_2 : b_1 \left(\text{or } \frac{a_1}{a_2} = \frac{b_2}{b_1} \right)$$

where a_1, a_2 are any two values of A and b_1, b_2 are the corresponding values of B.

Quick Test

Determine whether each of the following is true (T) or false (F).

1. Rate has units. (T / F)
2. Ratio is a comparison of quantities of different kinds. (T / F)
3. If $16a = 12b$, then $a : b = 4 : 3$. (T / F)
4. If $a : b : c = 10 : 4 : 5$, then $a : c = 2 : 1$. (T / F)

Fill in the Blanks

1. Determine whether a rate or a ratio should be used to relate the quantities in each of the following.

(a) A machine produces 80 LED bulbs hourly. ___________

(b) The height of Andy is twice the height of Billy. ___________

2. Candy produces vases at a rate of 5 pieces/hour. How long does it take for her to make 30 vases?

It takes ___________ hours for her to make 30 vases.

3. Coffee bean is sold at the price of \$0.3/g. How much should Mary pay when buying 400 g of coffee bean?

Mary should pay ___________ for buying 400 g of coffee bean.

4. The scale of a map is 1 : 30 000. Find the actual length in km of a bridge which is 1 cm on the map.

The actual length is ___________ .

5. Andy and Billy divide \$4000 in the ratio of 5 : 3. How much does Billy get?

Billy gets ___________ .

TSA **6.** The average speed of a car is 64 km/h. If the car travels 200 km, the time required for the car is ___________ minutes.

7. If X and Y are in inverse proportion, then $x_1y_1 =$ ___________, where x_1, x_2 are any two values of X and y_1, y_2 are the corresponding values of Y.

Exercise

Level 1

Express each of the following rates in the given units (**1 – 3**).

TSA **1.** A boy runs 330 m in 1 minute. ___________ m/s

2. The annual salary of a man is \$540 000. \$ ___________ /month

3. The cost of 4 oranges is \$10. \$ ___________ /piece

Simplify each of the following ratios (**4 – 7**).

4. 32 : 24

5. $\frac{1}{6} : \frac{5}{8}$

For HKDSE **33.** Consider the formula $\frac{5x+3y}{-x-3y}=3$.

(a) Make x the subject of the above formula.

(b) If $3x + 2y = 5$, find the values of x and y.

Crossover Mathematics

Famous Formulas

There are many formulas to describe theories or theorems in different areas. The following shows some examples.

A formula of GDP (Economics): $Y = C + I + G + (X - M)$

GDP stands for Gross Domestic Product （國民生產總值）. Briefly, it is how much a place produces in a period. In the formula, Y, C, I, G, X and M are GDP, consumption, investment, government spending, exports and imports respectively.

Formula of mass-energy equivalence (Physics): $E = mc^2$

It is a formula from special theory of relativity （狹義相對論） which was developed by Albert Einstein （愛因斯坦）. In the formula, E, m and c are energy, mass and speed of light respectively.

Glossary

algebraic fraction	代數分式	change of subject	主項變換
denominator	分母	formula	公式
method of substitution	代入法	numerator	分子
simplification	化簡	subject	主項